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# Gauge-Yukawa Unification<sup>†</sup>

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## Abstract

Gauge-Yukawa Unification (GYU) yields a functional relation among the gauge and Yukawa couplings, and it may follow from the usual Grand Unification if it is supplemented with some additional principles. Postulating the principles of finiteness and reduction of couplings, we have achieved Gauge-Yukawa Unification in various supersymmetric unified models leading, among other things, to interesting predictions on the top quark mass.

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The traditional way of reducing the number of the free parameters of the standard model (SM) is to require that the theory is more symmetric at higher energies. This approach has been applied, e.g., in GUTs with a certain success in the gauge and independently in the Yukawa sectors of the theory. However, this attractive principle has its limitation as it is well known that increasing the gauge symmetry of a GUT (e.g.,  $SO(10)$ ,  $E_6$ ,  $E_7$ ,  $E_8$ ) one does not necessarily increase the predictive power of the theory. This is because, to construct realistic models, one has to understand the breaking of these symmetries, which requires introducing additional free parameters in general. Alternatively, we suggest [1]-[3] that a natural gradual extension of the GUT idea, in prospect of increasing the predictability of the low energy parameters of the theory, is to attempt to relate the couplings of the gauge and Yukawa sectors, i.e., to achieve Gauge-Yukawa Unification (GYU). Searching for a symmetry that would provide GYU, one is led to consider  $N = 2$  supersymmetric theories [4], which however proved to have more serious phenomenological problems than the SM. The last comment holds also for superstring theories and composite models which could in principle lead to relations among the gauge and Yukawa couplings.

In our recent studies [1]-[3], we have considered the GYU which is based on the principles of reduction of couplings [7, 8, 2, 3] and also finiteness [9, 12, 1]. These principles, which are formulated in perturbation theory, are not explicit symmetry principles, although they might imply symmetries. The former principle is based on the existence of renormalization group invariant (RGI) relations among couplings which preserve perturbative renormalizability. Similarly, the latter one is based on the fact that it is possible to find RGI relations among couplings that keep finiteness in perturbation theory, even to all orders [12]. Applying these principles, one can relate the gauge and Yukawa couplings without introducing necessarily a symmetry, thereby improving the predictive power of a model. In what follows, we briefly outline the basic tool of this GYU scheme and its application to various models.

A RGI relation among couplings can be expressed in an implicit form

$$\Phi(g_1, \dots, g_N) = 0, \quad (1)$$

which has to satisfy the partial differential equation (PDE)  $\mu d\Phi/d\mu = \sum_{i=1}^N \beta_i \partial\Phi/\partial g_i = 0$ ,

where  $\beta_i$  is the  $\beta$ -function of  $g_i$ . There exist  $(N - 1)$  independent  $\Phi$ 's, and finding the complete set of these solutions is equivalent to solve the so-called reduction equations [7],

$$\beta_g \frac{dg_i}{dg} = \beta_i, \quad i = 1, \dots, N, \quad (2)$$

where  $g$  and  $\beta_g$  are the primary coupling and its  $\beta$ -function, and  $i$  does not include it. Using all the  $(N - 1)$   $\Phi$ 's to impose RGI relations, one can in principle express all the couplings in terms of a single coupling  $g$ . The complete reduction, which formally preserve perturbative renormalizability, can be achieved by demanding power series solution

$$g_i = \sum_{n=0} \kappa_i^{(n)} g^{2n+1}, \quad (3)$$

where the uniqueness of such a power series solution can be investigated at the one-loop level [7]. The completely reduced theory contains only one independent coupling with the corresponding  $\beta$ -function. In susy Yang-Mills theories with a simple gauge group, something more drastic can happen; the vanishing of the  $\beta$ -function to all orders in perturbation theory, if all the one-loop anomalous dimensions of the matter fields in the completely, uniquely reduced theory identically vanish [12].

This possibility of coupling unification is attractive, but it can be too restrictive and hence unrealistic. To overcome this problem, one may use fewer  $\Phi$ 's as RGI constraints. This is the idea of partial reduction [8, 2, 3], and the power series solution (3) becomes in this case

$$g_i = \sum_{n=0} \kappa_i^{(n)} (g_a/g) g^{2n+1}, \quad i = 1, \dots, N', \quad a = N' + 1, \dots, N. \quad (4)$$

The coefficient functions  $\kappa_i^{(n)}$  are required to be unique power series in  $g_a/g$  so that the  $g_a$ 's can be regarded as perturbations to the completely reduced system in which the  $g_a$ 's identically vanish. In the following, we would like to consider three different GYU models.

### A. Finite Unified Theory (FUT) based on $SU(5)$ [1]

This is a  $N = 1$  susy Yang-Mills theory based on  $SU(5)$  [11] which contains one **24**, four pairs of  $(\mathbf{5} + \overline{\mathbf{5}})$ -Higgses and three  $(\overline{\mathbf{5}} + \mathbf{10})$ 's for three fermion generations. The unique power series solution [1], which looks realistic as a first approximation, corresponds

to the Yukawa matrices without intergenerational mixing, and yields in the one-loop approximation

$$g_t^2 = g_c^2 = g_u^2 = \frac{8}{5}g^2, \quad g_b^2 = g_s^2 = g_d^2 = g_\tau^2 = g_\mu^2 = g_e^2 = \frac{6}{5}g^2, \quad (5)$$

where  $g_i$ 's stand for the Yukawa couplings. At first sight, this GYU seems to lead to unacceptable predictions of the fermion masses. But this is not the case, because each generation has an own pair of  $(\bar{\mathbf{5}} + \mathbf{5})$ -Higgses so that one may assume [11, 1] that after the diagonalization of the Higgs fields the effective theory is exactly MSSM, where the pair of its Higgs supermultiplets mainly stems from the  $(\mathbf{5} + \bar{\mathbf{5}})$  which couples to the third fermion generation. (The Yukawa couplings of the first two generations can be regarded as free parameters.) The predictions of  $m_t$  and  $m_b$  for various  $m_{\text{SUSY}}$  are given in table 1.

$m_{\text{SUSY}}$ [GeV]	$\alpha_3(M_Z)$	$\tan \beta$	$M_{\text{GUT}}$ [GeV]	$m_b$ [GeV]	$m_t$ [GeV]
200	0.123	53.7	$2.25 \times 10^{16}$	5.2	184.0
500	0.118	54.2	$1.45 \times 10^{16}$	5.1	184.4

**Table 1.** The predictions for  $m_{\text{SUSY}} = 200$  and 500 GeV for FUT.

## B. Asymptotically free Dimopoulos-Georgi-Sakai (DGS) Model [2]

The field content is minimal. Neglecting the Cabibbo-Kobayashi-Maskawa mixing, there are six Yukawa and two Higgs couplings at the beginning. We then require GYU to occur among the Yukawa couplings of the third generation and the gauge coupling. We also require the theory to be completely asymptotically free. In the one-loop approximation, the GYU yields  $g_{t,b}^2 = \sum_{m,n=1}^{\infty} \kappa_{t,b}^{(m,n)} h^m f^n g^2$ . ( $h$  and  $f$  are related to the Higgs couplings.)  $h$  is allowed to vary from 0 to 15/7, while  $f$  may vary from 0 to a maximum which depends on  $h$  and vanishes at  $h = 15/7$ . As a result, we obtain[2]

$$0.97 g^2 \lesssim g_t^2 \lesssim 1.37 g^2, \quad 0.57 g^2 \lesssim g_b^2 = g_\tau^2 \lesssim 0.97 g^2. \quad (6)$$

In table 2, we give some representative predictions.

$m_{\text{SUSY}}$ [GeV]	$g_t^2/g^2$	$g_b^2/g^2$	$\alpha_3(M_Z)$	$\tan \beta$	$M_{\text{GUT}}$ [GeV]	$m_b$ [GeV]	$m_t$ [GeV]
300	1.37	0.97	0.120	52.2	$1.9 \times 10^{16}$	5.2	182.8
300	0.97	0.57	0.120	47.7	$1.8 \times 10^{16}$	5.4	179.7
500	1.37	0.97	0.118	52.4	$1.43 \times 10^{16}$	5.1	182.7
500	0.97	0.57	0.118	47.7	$1.39 \times 10^{16}$	5.3	178.9

**Table 2.** The predictions of the asymptotically free DGS model

### C. Asymptotically non-free SUSY Pati-Salam (PS) Model [3]

This is a model without covering GUT [13], that is, there is no gauge coupling unification as it stands. The field content is [3]: three  $(\mathbf{4}, \mathbf{2}, \mathbf{1})$  and three  $(\bar{\mathbf{4}}, \mathbf{1}, \mathbf{2})$  under  $SU(4) \times SU(2)_L \times SU(2)_R$  for three fermion generations, a set of  $(\mathbf{4}, \mathbf{2}, \mathbf{1})$ ,  $(\bar{\mathbf{4}}, \mathbf{2}, \mathbf{1})$  and two  $(\mathbf{15}, \mathbf{1}, \mathbf{1})$  for Higgses that are responsible for the spontaneous symmetry breaking down to  $SU(3)_C \times SU(2)_L \times U(1)_Y$ , and also a set of  $(\mathbf{1}, \mathbf{2}, \mathbf{2})$ ,  $(\mathbf{15}, \mathbf{2}, \mathbf{2})$  and  $(\mathbf{1}, \mathbf{1}, \mathbf{1})$ . The singlet supermultiplet mixes with the right-handed neutrino supermultiplets at a high energy scale, while  $(\mathbf{15}, \mathbf{2}, \mathbf{2})$  is introduced to realize the Georgi-Jarlskog type ansatz for the fermion mass matrix.

In one-loop order, we first obtain the unification of the gauge couplings,

$$g_4^2 = \frac{8}{9} g_{2L}^2, \quad g_{2R}^2 = \frac{4}{5} g_{2L}^2. \quad (7)$$

In the Yukawa sector, we find

$$2.8 g_{2L}^2 \lesssim g_t^2 = g_b^2 = g_\tau^2 \lesssim 3.5 g_{2L}^2. \quad (8)$$

The typical predictions are presented in table 3.

$m_{\text{SUSY}}$ [GeV]	$g_t^2/g_{2L}^2$	$\alpha_3(M_Z)$	$\tan \beta$	$M_{\text{GUT}}$ [GeV]	$m_b$ [GeV]	$m_t$ [GeV]
500	2.8	0.129	61.2	$0.16 \times 10^{16}$	5.4	196.8
500	3.4	0.132	62.1	$0.17 \times 10^{16}$	5.4	198.3
1600	2.8	0.114	62.5	$0.07 \times 10^{16}$	4.8	192.7
1600	3.4	0.112	63.4	$0.06 \times 10^{16}$	4.7	193.3

**Table 3.** The predictions of the asymptotically non-free SUSY Pati-Salam model

In all of the analyses above, we have used the RG technique and regarded the GYU relations (5)-(8) as the boundary conditions holding at the unification scale  $M_{\text{GUT}}$ . We have assumed that it is possible to arrange the susy mass parameters along with the soft breaking terms in such a way that the desired symmetry breaking pattern really occurs, all the superpartners are unobservable at present energies, there is no contradiction with proton decay, and so forth. To simplify our numerical analysis we have also assumed a unique threshold  $m_{\text{SUSY}}$  for all the superpartners. Then we have examined numerically the evolution of the gauge and Yukawa couplings below  $M_{\text{GUT}}$  including the two-loop effects.

Concerning recent related studies by other authors, we would like to emphasize that our approach of dealing with asymptotically non-free theories [3] covers ref. [14] though the underlying idea might be different. In ref. [15], interesting RGI relations among the soft breaking parameters above the unification scale has been found. These relations are obtained on the close analogy of our approach presented here.

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